(1) (40 points) **Problem 3.13 (Messenger and Abtahi, modified)**

If a maximum power point tracker operates at 98% efficiency with a PV array that has the characteristics shown below, using the load curves as shown in the figure:

a) Determine the additional power available to each load when the array operates with solar irradiation of 1000 W/m²

b) For each load, determine the charge controller input current and output current when the array operates with solar irradiation of 1000 W/m²

c) Repeat a,b if the array operates with solar irradiation of 500 W/m²

(a) The maximum power point (MPP) for irradiation of 1000W/m² occurs at roughly

\[ V_{mp} \approx 90V \quad I_{mp} \approx 21A \quad \rightarrow \quad P_m \approx 1890W \]

For Load 1, the intersection of the load line with the PV array I-V curve occurs at approximately

\[ V_{L1} \approx 40V \quad I_{L1} \approx 25A \quad \rightarrow \quad P_{L1} \approx 1000W \]
For Load 2, the intersection of the load line with the PV array I-V curve occurs at approximately

\[ V_{L2} \approx 67V \quad I_{L2} \approx 24A \quad \rightarrow \quad P_{L2} \approx 1600W \]

With 98% MPPT efficiency, the available power from the array is roughly

\[ P_{\text{avail}} \approx 1890W \cdot 0.98 = 1850W \]

Therefore, the additional power delivered to the loads is

Load 1: \[ \Delta P_1 = P_{\text{avail}} - P_{L1} = 850W \]

Load 2: \[ \Delta P_2 = P_{\text{avail}} - P_{L2} = 250W \]

(b) For Load 1:

\[ I_{\text{in}} = I_{\text{mp}} = 21A \]

\[ P_{\text{out}} = V_{\text{out}}I_{\text{out}} = 1850W \]

By looking at the Load 1 I-V curve and its intersection with the constant power hyperbola

\[ I = \frac{1850}{V} \]

We see that

\[ I_{\text{out1}} \approx 28.5A \quad V_{\text{out1}} \approx 65V \]
For Load 2, again the graphical solution gives:

\[ I_{out2} \approx 25A \quad V_{out2} \approx 75V \]

(c) For the lower illumination, referring to the following graph:

\[ V_{mp} \approx 90V \quad I_{mp} \approx 10A \implies P_m \approx 900W \]

And the intersection points yield:

\[ V_{L1} \approx 14V \quad I_{L1} \approx 12.5A \implies P_{L1} \approx 175W \]
\[ V_{L2} \approx 53V \quad I_{L2} \approx 12A \implies P_{L2} \approx 640W \]

The available power is:

\[ P_{avail} = 900W \times 0.98 = 880W \]

\[ \Delta P_1 = P_{avail} - P_{L1} = 705W \]
\[ \Delta P_2 = P_{avail} - P_{L2} = 240W \]
For both loads:

\[ I_{in} \approx 10\,A \]

And from the graph:

\[ I_{out1} \approx 24\,A \quad V_{out1} \approx 38\,V \]
\[ I_{out2} \approx 16\,A \quad V_{out2} \approx 55\,V \]
(2) (30 points) Problem 3.14 (Messenger and Abtahi)

A battery charge controller incorporates an MPPT to optimize the charging of the batteries. Assume the maximum power voltage ($V_{mp}$) of a PV array is 99V and the bulk charge voltage level for a 48V vented lead-acid battery bank is 56V. If the conversion efficiency of the MPPT is 98%, estimate the percentage increase in charge delivered to the batteries under the conditions over that which would be delivered by a controller that causes the array to operate at 56V rather than 99V. You may assume the PV array consists of three modules connected in series, that $V_{mp}$ is independent of irradiance levels, and each module has $V_{oc} = 42V, I_{sc} = 6.25A, V_{mp} = 33V,$ and $I_{mp} = 5.76A$

The power delivered to the batteries using the MPPT is given by:

$$P_{batt,m} = \eta V_{mp}I_{mp} = (0.98)(99V)(5.76A) = 559W$$

If the battery is connected to the PV array without MPPT, the power sent to the batteries would be:

$$P_{batt} = V_{ch}I_{ch} = (56V)(6.25A) = 350W$$

We have assumed that the battery voltage is the bulk charge voltage level, and the battery current is a bit higher than $I_{mp}$, so we used $I_{sc}$. The percentage gain in charge is equal to the percentage gain in power, so:

$$\frac{\Delta Q}{Q} = \frac{\Delta P}{P} = \frac{P_{batt,m} - P_{batt}}{P_{batt}} = \frac{559 - 350}{350} = 0.60 = 60\%$$
(3) (30 points) A Pulse Width Modulation (PWM) problem

Calculate the PWM sequence needed to produce this waveform:

A sinusoidal increase over the same time period \((0 < t < 1\, \text{s})\), and same voltage range \((0 < v_b(t) < 10\, \text{V})\), using 10 pulses

\[
v_b(t) = 10 \sin\left(\frac{\pi t}{2}\right) \quad \text{(in volts)}
\]

\[0 \leq t \leq 1 \quad \text{(in seconds)}\]

Plot the ideal voltage (analytical expression), the real voltage, and the pulses versus time

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It is sensible to examine the 10 segments in a general fashion, using an index to identify each segment. So the average value in any segment is:

\[
\langle v \rangle_k = \frac{1}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} v_b(t) \, dt = \frac{1}{\Delta t} \int_{t_k}^{t_{k+1}} \sin\left(\frac{\pi t}{2}\right) \, dt
\]

\[
= \frac{10}{\Delta t} \frac{\pi}{2} \left[ \cos\left(\frac{\pi t}{2}\right) \right]_{t_k}^{t_{k+1}} = \frac{200}{\pi} \left[ \cos\left(\frac{\pi t_{k+1}}{2}\right) - \cos\left(\frac{\pi t_k}{2}\right) \right]
\]

and
The following table lists the relevant values:

\[
D_k = \frac{\langle v \rangle_k}{V_{\text{max}}} = \frac{\langle v \rangle_k}{10} = \frac{20}{\pi} \left[ \cos\left(\frac{\pi t_{k+1}}{2}\right) - \cos\left(\frac{\pi t_k}{2}\right) \right]
\]

And the voltages and pulses are plotted below:
Bonus problem:

a. A linear increase in voltage from $t = 0$ to $t = 1$ s, using 10 pulses

\[ V_a(t) = 10t \]  \hspace{1cm} \text{(in volts)}

\[ 0 \leq t \leq 1 \]  \hspace{1cm} \text{(in seconds)}

Calculating the average value in any segment is:

\[
\langle v \rangle_k = \frac{1}{t_{k+1} - t_k} \int_{t_k}^{t_{k+1}} v_a(t) \, dt = \frac{1}{t_{k+1} - t_k} 10 \int_{t_k}^{t_{k+1}} t \, dt
\]

\[
= \frac{10}{t_{k+1} - t_k} \left[ \frac{t^2}{2} \right]_{t_k}^{t_{k+1}} = \frac{5}{t_{k+1} - t_k} \left[ t_{k+1}^2 - t_k^2 \right] = \frac{5}{t_{k+1} - t_k} \left( t_{k+1} - t_k \right) \left( t_{k+1} + t_k \right)
\]

Simplifying this:

\[
\langle v \rangle_k = 5(t_{k+1} + t_k)
\]

Substituting

\[ k = 0, 1, 2, \ldots 9 \]

the following results are found:
Furthermore, the duty cycle is the ratio of the width of the pulse to the width of the segment, or as shown in class:

\[ D_k = \frac{\langle v \rangle_k}{V_{\text{max}}} \]

or in this case:

\[ D_k = \frac{\langle v \rangle_k}{V_{\text{max}}} = \frac{5(t_{k+1} + t_k)}{10} = \frac{(t_{k+1} + t_k)}{2} \]

So we have,

<table>
<thead>
<tr>
<th>( t_k \rightarrow t_{k+1} )</th>
<th>( t_{k+1} + t_k \text{ (s)} )</th>
<th>( v_k \text{ (V)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 – 0.1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>0.1 – 0.2</td>
<td>0.3</td>
<td>1.5</td>
</tr>
<tr>
<td>0.2 – 0.3</td>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>.....</td>
<td>.....</td>
<td>.....</td>
</tr>
<tr>
<td>0.7 – 0.8</td>
<td>1.5</td>
<td>7.5</td>
</tr>
<tr>
<td>0.8 – 0.9</td>
<td>1.7</td>
<td>8.5</td>
</tr>
<tr>
<td>0.9 – 1.0</td>
<td>1.9</td>
<td>9.5</td>
</tr>
</tbody>
</table>

The voltages and pulses are plotted here:
PWM sequence for 10t